International Journal of Control, Automation and Systems 16(5) (2018) 2264-2272
http://dx.doi.org/10.1007/s12555-017-0705-y
ISSN:1598-6446 eISSN:2005-4092
http://www.springer.com/12555

Desired Compensation Adaptive Robust Control of Electrical-optical Gyro-stabilized Platform with Continuous Friction Compensation Using Modified LuGre Model

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Abstract: Continuous friction compensation along with other modeling uncertainties is concerned in this paper, to result in a continuous control input, which is more suitable for controller implementation. To accomplish this control task, a novel continuously differentiable nonlinear friction model is synthesized by modifying the traditional piecewise continuous LuGre model, then a desired compensation version of the adaptive robust controller is proposed for precise tracking control of electrical-optical gyro-stabilized platform systems. As a result, the adaptive compensation and the regressor in the proposed controller will depend on the desired trajectory and on-line parameter estimates only. Hence, the effect of measurement noise can be reduced and then high control performance can be expected. Furthermore, the proposed controller theoretically guarantees an asymptotic output tracking performance even in the presence of modeling uncertainties. Extensively comparative experimental results are obtained to verify the effectiveness of the proposed control strategy.

Keywords: Adaptive robust control, desired compensation friction compensation, three-axis electrical-optical gyro-stabilized platform (TEOGSP).

1. INTRODUCTION

Three-axis electrical-optical gyro-stabilized platform (TEOGSP) equipment has been used widely in the flight military area such as precision striking of fire-control weapons, warning and trajectory predicted of missile and landing [1]. When the aircraft vibrates, even a small imbalance distance existing in the load can lead to a large mass imbalance torque. These disturbances existing in the TEOGSP equipment integrate a characteristic of nonlinear and modelling uncertainties [2, 3], resulting in a serious degradation of control accuracy [4, 5]. For high precision motion performance, the friction problem is one of the significant limitations since friction is a very complicated phenomenon. To predict the underlying nonlinear friction for controller compensation, many friction models are proposed in [6, 7]. Among of them, the LuGre model in [8] has been widely employed for controller design, due to its simple structure and capability to capture most of the observed frictional behaviors. In order to adapt to the backstepping controller designing process, some novel continuously differentiable modified LuGre models were proposed in [9, 10] is utilized to develop a continuous friction compensation law to handle other structured uncertainties and unstructured uncertainties for a motion system directly driven by a dc motor. This new static friction model proposed in [11, 12] captures the major effects of friction without involving discontinuous or piecewise continuous functions.

Though all above mentioned control strategies can effectively solve the control problems in uncertain nonlinear systems, the effects of measurement noise via full state feedback are not specifically taken into consideration. A good alternative way of attenuating the effect of measurement noise is the desired compensation adaptation law which was first proposed in [13] for the trajectory tracking control of robot manipulators. The strategy is applied to the precision motion control of linear motor drive systems in [14] and excellent tracking performance is obtained in experiments. To address the problem of unmatched model uncertainties, a general framework on the desired compensation adaptive robust control (DCARC) backstepping design procedure [15] will be used. However, for uncertain nonlinear systems with unmatched modeling uncertainties and input nonlinearities, the desired values of the intermediate state variables cannot be predetermined based on the methods in [15–17]. In [18], a general way of constructing the desired values of intermediate state variables was theoretically discussed to overcome the design difficulties associated with unmatched modeling uncertainties for a

Manuscript received November 13, 2017; revised January 27, 2018 and April 10, 2018; accepted April 11, 2018. Recommended by Editor Hamid Reza Karimi. This work was supported in part by the NUPTSF (Grant No. NY215053).

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class of uncertain nonlinear systems. Different from the linear motor drive system in [18], the model-based desired compensation adaptive robust controller (DCARC) is also developed in [19] to reduce the overall control effort/chattering and the noise sensitivity problem in application, and thus the tracking accuracy can be further improved.

Motivated by the above observations, this paper employs a new continuously differentiable friction model proposed in [12] to develop a continuous friction compensation law that cancels the majority of nonlinear friction in the systems, while in conjunction with the desired compensation version of the adaptive robust control approach [20] to handle other unconsidered disturbances for electrical-optical gyro-stabilized platform systems. In our proposed controller, the model based desired compensation is employed to reduce the overall control chattering and the noise sensitivity in application, and thus, the tracking accuracy can be further improved.

The proposed approach in this paper presents an asymptotic output tracking performance in the presence of modeling uncertainties with adequate friction compensation, which is vital for high-precision motion control. Moreover, the desired compensation technique utilized in our controller has the unique feature that the model compensation law depends on the reference trajectory only. Furthermore, the interaction between the parameter adaptation and the robust control law is reduced, which may facilitate the controller gain tuning process considerably. Comparative experimental results are obtained for verification on TEOGSP system.

2. PROBLEM FORMULATION AND DYNAMIC MODELS

2.1. Modeling of the TEOGSP system

For an TEOGSP, the plant is composed of three parts: dc brushed torque motor, gear transmission, and gimbal system, as shown in Fig. 1. The gears and gimbal are usually regarded as rigid bodies. Based on this, the dynamic model of an TEOGSP is summarized as

$$\begin{cases} L\,di/dt + iR + K_E N\omega_l = u, \\ (N^2J_m + J_l)\,\dfrac{d\omega_l}{dt} = NK_F i - T_d - T_f, \\ \dfrac{d\theta_l}{dt} = \omega_l, \end{cases} \tag{1}$$

where u is the motor voltage, i the motor current, L the armature inductance, R the resistor, N the gear transmission ratio, K_F the motor torque coefficient, J_m the motor inertia moment, J_l the gimbal inertia moment, θ_l the attitude angle, ω_l the angular rate, K_E is the electromotive force coefficient, T_f represents the nonlinear friction, T_d represents the unconsidered dynamic as well as external disturbance.

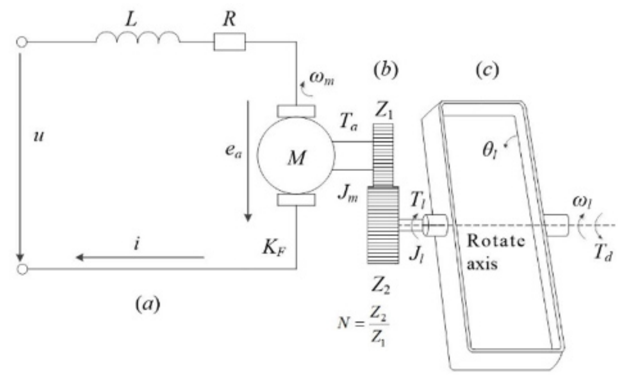

Fig. 1. System model for an STEOGSP (a) DC brushed motor (b) Gear transmission (c) Gimbal system.

There have been many friction models proposed in [10], but they cannot be used in backstepping controller design because of the discontinuous property. The friction force T_f can be described by

$$T_f(t) = \sigma_0 z + \sigma_1 \dot{z} + \sigma_2 \omega, \tag{2}$$

where σ_0, σ_1 and σ_2 represent the friction force parameters which can be physically interpreted as stiffness coefficient, z represents the unmeasurable internal friction state, and its characteristic is governed by

$$\begin{aligned} \dot{z} &= \omega\left[1 - \frac{1}{g(\omega)} z\right], \\ g(\omega) &= (f_s - f_c)\left[\tanh(\lambda_1\omega) - \tanh(\lambda_2\omega)\right] \\ &\quad + l_2\tanh(\lambda_3\omega) + l_3\omega, \end{aligned} \tag{3}$$

where f_c and f_s represent the levels of the normalized Coulomb friction and stiction force respectively; l_1, l_2 and l_3 represent different friction levels; and λ_1, λ_2, λ_3 denote various shape coefficients to approximate various friction effects.

The state variables of the TEOGSP system are chosen as x=$[x_1\ x_2\ x_3]^T$=$[\theta_l\ \omega_l\ NK_F i/(N^2J_m$+$J_l)]^T$. Integrating the developed friction and the motor model, the dynamic of the entire system can be formulated as

$$\begin{cases} \dot{z} = x_2 - N(x_2)z, \\ \dot{x}_1 = x_2, \\ \dot{x}_2 = x_3 - \dfrac{\sigma_0}{N^2J_m + J_l} z + \dfrac{\sigma_1}{N^2J_m + J_l} N(x_2) z \\ \qquad - \dfrac{\sigma_1 + \sigma_2}{N^2J_m + J_l} x_2 - \dfrac{T_d}{N^2J_m + J_l}, \\ \dot{x}_3 = -\dfrac{NK_F R}{L(N^2J_m + J_l)} x_3 - \dfrac{NK_F K_E}{L(N^2J_m + J_l)} x_2 \\ \qquad + \dfrac{NK_F}{L(N^2J_m + J_l)} u. \end{cases} \tag{4}$$

Define the lumped disturbance and unknown parameters as

$$\theta_1=\frac{\sigma_0}{N^2J_m+J_l},\ \ \theta_2=\frac{\sigma_1}{N^2J_m+J_l},\ \ \theta_3=\frac{\sigma_1+\sigma_2}{(N^2J_m+J_l)K_F},$$
$$\theta_4=\frac{d_n}{N^2J_m+J_l},\ \ \theta_5=\frac{NK_F}{(N^2J_m+J_l)L},\ \ \tilde{d}=\frac{T_d-d_n}{N^2J_m+J_l},$$
$$\theta_6=\frac{NK_FK_E}{(N^2J_m+J_l)L},\ \ \theta_7=\frac{NK_FR}{(N^2J_m+J_l)L}, \tag{5}$$

where d_n is the constant part of T_d and $\tilde{d}$ is the time-varying part. We assume that $\theta\in\Omega_\theta=\{\theta:\theta_{\min}\le\theta\le\theta_{\max}\}$ and $|\tilde{d}(t)|\le\delta_1(t)$, where $\theta_{\min}=[\theta_{1\min},\dots,\theta_{7\min}]^T$, $\theta_{\max}=[\theta_{1\max},\dots,\theta_{7\max}]^T$, and $\delta_1(t)$ is known.

Thus the state-space equation can be formulated as

$$\begin{cases}\dot{z}=x_2-N(x_2)z,\\ \dot{x}_1=x_2,\\ \dot{x}_2=x_3-\theta_1z+\theta_2N(x_2)z-\theta_3x_2-\theta_4-\tilde{d},\\ \dot{x}_3=\theta_5u-\theta_6x_2-\theta_7x_3.\end{cases} \tag{6}$$

2.2. Projection mapping and parameter adaptation

Let $\hat{\theta}$ denote the estimate of θ and $\tilde{\theta}$ the estimation error (i.e., $\tilde{\theta}=\hat{\theta}-\theta$). A discontinuous projection can be defined as

$$\mathrm{Proj}_{\hat{\theta}_i}(\bullet_i)=\begin{cases}0 & \text{if } \hat{\theta}_i=\theta_{i\max} \text{ and } \bullet_i>0,\\ 0 & \text{if } \hat{\theta}_i=\theta_{i\min} \text{ and } \bullet_i<0,\\ \bullet_i & \text{otherwise,}\end{cases} \tag{7}$$

where $i=1,\dots,7$. By using an adaptation law given by

$$\dot{\hat{\theta}}=\mathrm{Proj}_{\hat{\theta}}(\Gamma\tau) \text{ with } \theta_{\min}\le\hat{\theta}(0)\le\theta_{\max}, \tag{8}$$

where Γ is a positive diagonal adaptation rate matrix, and τ is an adaptation function to be synthesized later; for any adaption function τ, the projection mapping used in (10) guarantees

$$\textbf{(P1)}\ \hat{\theta}\in\Omega_{\hat{\theta}}=\left\{\hat{\theta}:\theta_{\min}\le\hat{\theta}\le\theta_{\max}\right\},$$
$$\textbf{(P2)}\ \tilde{\theta}^T[\Gamma^{-1}\mathrm{Proj}_{\hat{\theta}}(\Gamma\tau)-\tau]\le 0,\ \ \forall\tau. \tag{9}$$

In order to handle different characteristics of z, robust observers with projection type modifications were proposed in [16], based on the dual-observer structure [17]:

$$\dot{\hat{z}}_1=\mathrm{Proj}_{\hat{z}_1}(\eta_1),\ \ \dot{\hat{z}}_2=\mathrm{Proj}_{\hat{z}_2}(\eta_2), \tag{10}$$

where $\hat{z}_1$, $\hat{z}_2$ are estimates of the unmeasurable friction state z; η_1 and η_2 are learning functions to be synthesized later.

3. CONTROLLER DESIGN

3.1. Adaptive robust controller (ARC) design

According to these explicit nonlinear models, an adaptive robust backstepping control technique with carefully chosen Lyapunov functions is proposed. The design procedure is as follows:

Define a set of switching-function-like quantities as

$$e_2=\dot{e}_1+k_1e_1=x_2-x_{2eq},\ \ e_3=x_3-\alpha_2, \tag{11}$$

where $e_1=x_1-x_{1d}(t)$ is the output tracking error; k_1 is a positive feedback gain.

The robust control law α_2 consists of two terms given by

$$\begin{cases}\alpha_2=\alpha_{2a}+\alpha_{2s},\\ \alpha_{2a}=\ddot{x}_{1d}+\hat{\theta}_1\hat{z}_1-\hat{\theta}_2N(x_2)\hat{z}_2+\hat{\theta}_3x_2+\hat{\theta}_4,\\ \alpha_{2s}=\alpha_{2s1}+\alpha_{2s2},\ \ \alpha_{2s2}=-k_{2s}e_2=-\dfrac{h_1+1}{4\varepsilon_1}e_2,\\ \alpha_{2s1}=-k_2e_2-g_2\left\|\Gamma\varphi_2\right\|^2e_2,\\ \varphi_2=[-\hat{z}_1,N(x_2)\hat{z}_2,-x_2,-1,0,0,0]^T,\end{cases} \tag{12}$$

where k_2 is a positive constant and Γ is a positive definite constant diagonal matrix, ε_1 is any positive scalar and h_1 is any smooth function satisfying the following conditions

$$h_1\ge[||\theta_M||||\varphi_2||+\theta_{1M}z_M+\theta_{2M}N(x_2)z_M]^2, \tag{13}$$

in which $\theta_M=\theta_{\max}-\theta_{\min}$, $\theta_{1M}=\theta_{1\max}-\theta_{1\min}$, $\theta_{2M}=\theta_{2\max}-\theta_{2\min}$, $z_M=z_{\max}-z_{\min}$.

The following actual control law u is proposed:

$$\begin{cases}u=u_a+u_s,\\ u_a=\dfrac{1}{\hat{\theta}_5}(\hat{\theta}_6x_2+\hat{\theta}_7x_3+\dot{\alpha}_2+\dfrac{\partial\alpha_2}{\partial x_1}x_2+\dfrac{\partial\alpha_2}{\partial x_2}x_3),\\ u_s=\dfrac{1}{\theta_{5\min}}(u_{s1}+u_{s2}),\ \ u_{s2}=-k_{s3}e_3=-\dfrac{h_2+1}{4\varepsilon_2}e_3,\\ u_{s1}=-k_{3s}e_3k_{3s}=k_3+d_3\left\|\dfrac{\partial\alpha_2}{\partial\hat{\theta}}\right\|^2+g_3\left\|\Gamma\varphi_3\right\|^2,\\ \varphi_3=[0,0,0,0,u_a,-x_2,-x_3]^T,\end{cases} \tag{14}$$

where k_3 is a positive constant, ε_2 is any positive scalar and h_2 is any smooth function satisfying the following conditions

$$h_2\ge||\theta_M||^2\left(||\varphi_3||^2_{\max}+\left|\frac{\partial\alpha_2}{\partial\alpha_2}\right|^2_{\max}||\varphi_3||^2_{\max}\right)$$
$$+\delta_1^2+\left|\frac{\partial\alpha_2}{\partial\alpha_2}\right|^2_{\max}\delta_1^2. \tag{15}$$

Theorem 1: Let the parameter estimates be updated by the projection type adaptation law (8) in which τ is chosen as $\tau=\varphi_2e_2+\left(\varphi_3-\frac{\partial\alpha_2}{\partial x_2}\varphi_2\right)e_3$, the projection type state observation (10) and learning functions given by

$$\eta_1=x_2-N(x_2)\hat{z}_1-\gamma_1e_2,$$

$$\eta_2 = x_2 - N(x_2)\hat{z}_2 + \gamma_2 N(x_2) e_2, \tag{16}$$

where γ_1 and γ_2 are learning gains.

If the control parameters g_j, $j = 2, 3$, are selected such that $g_j > \frac{1}{2d_3}$ and the matrix Λ defined below is positive definite

$$\Lambda = \begin{bmatrix} k_1 & -1/2 & 0 \\ -1/2 & k_2 & -\kappa_1/2 \\ 0 & -\kappa_1/2 & \kappa_2 \end{bmatrix}, \tag{17}$$

where

$$\kappa_1 = 1 + \left|\frac{\partial \alpha_2}{\partial x_2}\right| \left(k_{2s} + \frac{1}{4\varepsilon_3}\right), \kappa_2 = k_3 + \frac{\partial \alpha_2}{\partial x_2}, \tag{18}$$

then the proposed control law (14) guarantees that

A) In general, all signals are bounded. Furthermore, the positive definite function V defined by

$$V(t) = \frac{1}{2}e_1^2 + \frac{1}{2}e_2^2 + \frac{1}{2}e_3^2 \tag{19}$$

is bounded by

$$\begin{aligned} V(t) \le & V(0)\exp(-\kappa t) \\ & + \varepsilon \frac{1 + ||\delta(t)||_\infty}{\kappa}[1 - \exp(-\kappa t)], \end{aligned} \tag{20}$$

where $\varepsilon = \varepsilon_1 + \varepsilon_2$, and $\kappa = 2\lambda_{\min}(\Lambda)$, $\delta(t) = \max\{\delta_1^2(t))\}$.

B) If after a finite time t_0, $\tilde{d} = 0$, in addition to results in A), asymptotic output tracking is also achieved, i.e., $e_1 \to 0$ as $t \to \infty$.

Proof of Theorem 1: If the system exists time-variant modeling errors, noting that $\theta_5/\theta_{5\min} > 1$, based on the condition of h_1 and h_2, the time derivative of V satisfies

$$\begin{aligned} \dot{V} \le & -e^T \Lambda e - g_2 \|\Gamma \phi_2\|^2 e_2^2 - d_3 \left\|\frac{\partial \alpha_2}{\partial \hat{\theta}}\right\|^2 e_3^2 \\ & - g_3 \|\Gamma \phi_3\|^2 e_3^2 + \varepsilon + \varepsilon \delta_1^2(t) - \frac{\partial \alpha_2}{\partial \hat{\theta}} \dot{\hat{\theta}} e_3 \\ & - \gamma_1^{-1} \theta_1 N(x_2) \tilde{z}_1^2 - \gamma_2^{-1} \theta_2 N(x_2) \tilde{z}_2^2, \end{aligned} \tag{21}$$

where $e = [e_1 \quad e \quad e_3]^T$.

Hence, if g_j, $j = 2, 3$, are selected to satisfy the conditions $g_j > \frac{1}{2d_3}$, it can be inferred that

$$-\frac{\partial \alpha_2}{\partial \hat{\theta}} \dot{\hat{\theta}} e_3 \le \left\|\frac{\partial \alpha_2}{\partial \hat{\theta}} d_3\right\|^2 e_3^2 + \sum_{j=2}^{3} \|g_j \Gamma \phi_j\|^2 e_j^2. \tag{22}$$

Combining (21) and (22), it can be obtained that

$$\dot{V} \le -\kappa V + \varepsilon + \varepsilon \delta(t). \tag{23}$$

Thus, e is bounded which means the state x is bounded. From property P1 in (9), all estimated parameters are bounded, thus the control input u is bounded. That proves all signals in the closed loop systems are bounded.

Now for B), defining a Lyapunov function as

$$\begin{aligned} V_o = & \frac{1}{2}e_1^2 + \frac{1}{2}e_2^2 + \frac{1}{2}e_3^2 \\ & + \frac{1}{2}\tilde{\theta}^T \Gamma^{-1} \tilde{\theta} + \frac{1}{2}\theta_1 \gamma_1^{-1} \tilde{z}_1^2 + \frac{1}{2}\theta_2 \gamma_2^{-1} \tilde{z}_2^2. \end{aligned} \tag{24}$$

Noting the condition $\tilde{d} = 0$, and combing the proof of conclusion A, the time derivative of V_0 satisfies

$$\dot{V}_o \le -e^T \Lambda e \le -\lambda_{\min}(\Lambda)(e_1^2 + e_2^2 + e_3^2) = -W. \tag{25}$$

Therefore, $V_o \in L_\infty$ and $W \in L_2$. Since all signals are bounded, from (25), it is easy to check that $\dot{W}$ is bounded and thus uniformly continuous. By Barbalat's lemma, $W \to 0$ as $t \to \infty$ [16], which leads to B) of Theorem 1. □

Remark 1: As done in [17], we just simply choose k_1, k_2, and k_3 large enough in experiments while without worrying about the specific prerequisites. By doing so, prerequisites (17) will be satisfied for certain sets of values of k_1, k_2, and k_3, at least locally around the desired trajectory to be tracked.

3.2. Desired compensation ARC controller design

DCARC uses desired trajectory signals to form regressor, which has been shown to outperform ARC in terms of all indexes [17]. Following the same design procedure as in [19], a DCARC law using the proposed friction model is also constructed as follows.

As shown in [19], by applying mean value theorem, $\tilde{N}$ satisfies the following inequalities:

$$|\tilde{N}| = |\theta_2 z(N(x_2) - \theta_2 N(\dot{x}_{1d}))| \le c_1 |e_1| + c_2 |e_2|, \tag{26}$$

where c_1 and c_2 are some positive constants.

By referring to [14], the robust control law α_2 consists of two terms given by

$$\begin{cases} \alpha_2 = \alpha_{2a} + \alpha_{2s}, \\ \alpha_{2a} = \ddot{x}_{1d} + \hat{\theta}_1 \hat{z}_1 - \hat{\theta}_2 N(\dot{x}_{1d}) \hat{z}_2 + \hat{\theta}_3 \dot{x}_{1d} + \hat{\theta}_4, \\ \alpha_{2s} = \alpha_{2s1} + \alpha_{2s2}, \alpha_{2s1} = -k_{2s} e_2, \\ k_{2s} \ge k_2 + \beta_2 ||\Gamma_2 \varphi_{2d}||^2, \\ \alpha_{2s2} = -\dfrac{h_3 + 1}{2\varepsilon_3} e_2, \\ \varphi_{2d} = [-\hat{z}_1, N(\dot{x}_{1d})\hat{z}_2, -\dot{x}_{1d}, -1, 0, 0, 0]^T, \end{cases} \tag{27}$$

where k_{2s} is a positive constant feedback gain and β_2 is a positive definite constant, k_2 is any positive scalar and h_3 is any smooth function satisfying the following conditions

$$h_3 \ge ||\theta_M||^2 ||\varphi_{2d}||^2 + \delta_1^2. \tag{28}$$

The actual control input u is proposed:

$$\begin{cases} u = u_a + u_s, \\ u_a = \dfrac{1}{\hat{\theta}_5}\left(\hat{\theta}_6\dot{x}_{1d} + \hat{\theta}_7 x_{3d} + \dfrac{\partial\alpha_2}{\partial x_1}\dot{x}_{1d} + \dfrac{\partial\alpha_2}{\partial x_2}\ddot{x}_{1d}\right), \\ x_{3d} = \ddot{x}_{1d} + \hat{\theta}_1\hat{z}_1 - \hat{\theta}_2 N(\dot{x}_{1d})\hat{z}_2 + \hat{\theta}_3\dot{x}_{1d} + \hat{\theta}_4, \\ u_s = \dfrac{1}{\hat{\theta}_{5\min}}(u_{s1} + u_{s2}), \;\; u_{s2} = -k_{s3}e_3 = -\dfrac{1+h_4}{4\varepsilon_4}e_3, \\ u_{s1} = -k_{3s}z_3, \;\; k_{3s} \geq k_3 + d_2\left\|\dfrac{\partial\alpha_2}{\partial\hat{\theta}}\right\|^2 + \beta_3\|\Gamma_2\phi_3\|^2, \\ \varphi_{3d} = \left[\begin{array}{ccccccc} 0 & 0 & 0 & 0 & u_a & -\dot{x}_{1d} & -x_{3d}\end{array}\right], \end{cases} \tag{29}$$

where k_{3s}, k_3 are positive constants, d_2, β_3 are positive definite constants and ϕ_3 is a regressor to be specified later, h_4 is any smooth function satisfying the following conditions

$$h_4 \geq ||\theta_M||^2\left(||\varphi_{3d}||^2_{\max} + \left|\frac{\partial\alpha_2}{\partial\alpha_2}\right|^2_{\max}||\varphi_{3d}||^2_{\max}\right) + \delta_1^2 + \left|\frac{\partial\alpha_2}{\partial\alpha_2}\right|^2_{\max}\delta_1^2. \tag{30}$$

Theorem 2: Let the parameter estimates be updated by the projection type adaptation law (8) in which τ is chosen as

$$\tau = \phi_{2d}z_2 + \phi_3 z_3, \;\; \phi_3 = \varphi_{3d} - \frac{\partial\alpha_2}{\partial x_2}\varphi_{2d}. \tag{31}$$

The projection type state observation (10) and learning functions given by

$$\begin{aligned} \eta_1 &= x_2 - N(x_2)\hat{z}_1 - \gamma_1 e_2, \\ \eta_2 &= x_2 - N(x_2)\hat{z}_2 + \gamma_2 N(x_2)e_2, \end{aligned} \tag{32}$$

where γ_1 and γ_2 are learning gains.

If the control parameters β_j, $j = 2, 3$, are selected such that $\beta_j > \frac{1}{2d_2}$ and the feedback gains k_1, k_2, k_3 are chosen large enough such that the matrix defined below is positive definite

$$\Lambda_1 = \begin{bmatrix} k_1 & -\gamma_3/2 & -\gamma_5/2 \\ -\gamma_3/2 & \gamma_4 & -\gamma_6/2 \\ -\gamma_5/2 & -\gamma_6/2 & \gamma_7 \end{bmatrix}, \tag{33}$$

where

$$\begin{aligned} \gamma_3 &= 1 + k_1(k_1 + \theta_{3\max}) + c_1, \;\; \gamma_4 = k_2 - k_1 - \theta_{3\max} - c_2, \\ \gamma_6 &= 1 + \left(\theta_{7\max} + \left|\frac{\partial\alpha_2}{\partial x_2}\right|\right)\left(k_{2s} + \frac{1}{4\varepsilon_3}\right) + \theta_{6\max} \\ &\quad + (\theta_{3\max} + c_2)\left|\frac{\partial\alpha_2}{\partial x_2}\right|, \\ \gamma_7 &= k_3 + \theta_{7\min} + \frac{\partial\alpha_2}{\partial x_2}, \end{aligned}$$

$$\gamma_5 = k_1\theta_{6\max} + k_1\left|\frac{\partial\alpha_2}{\partial x_2}\right|\theta_{3\max} + c_1\left|\frac{\partial\alpha_2}{\partial x_2}\right|. \tag{34}$$

Then, the proposed control law (29) guarantees that

A) In general, all closed loop signals are bounded. In general, all signals are bounded. Furthermore, the positive definite function

$$V_a = \frac{1}{2}e_1^2 + \frac{1}{2}e_2^2 + \frac{1}{2}e_3^2 \tag{35}$$

is bounded by

$$V_a \leq V_a(0)\exp(-\kappa t) + \varepsilon\frac{1 + ||\delta(t)||^2_\infty}{\kappa}[1 - \exp(-\kappa t)], \tag{36}$$

where $\kappa = \lambda_{\min}(\Lambda_1)$ is the exponentially converging rate. $\varepsilon = \varepsilon_3 + \varepsilon_4$.

B) If after a finite time t_0, $\tilde{d} = 0$, in addition to results in A), asymptotic output tracking is also achieved, i.e., $e_1 \to 0$ as $t \to \infty$.

Proof of Theorem 2: According to the deductive method in the proof of Theorem 1, it can be obtained that

$$\dot{V}_a \leq -\lambda V_a + \varepsilon + \varepsilon\delta_1^2(t), \tag{37}$$

which leads to (36) and the results in A) of Theorem 2 can be inferred.

Now go to the proof of conclusion B of Theorem 2. Choose a positive definite function V_b as

$$V_b = V_a + \frac{1}{2}\tilde{\theta}^T\Gamma^{-1}\tilde{\theta} + \frac{1}{2}\theta_1\gamma_1^{-1}\tilde{z}_1^2 + \frac{1}{2}\theta_2\gamma_2^{-1}\tilde{z}_2^2. \tag{38}$$

Noting the condition $\tilde{d} = 0$, and combing the proof of conclusion A), the time derivative of V_b satisfies

$$\dot{V}_b \leq -e^T\Lambda_1 e \leq -\lambda_{\min}(\Lambda_1)(e_1^2 + e_2^2 + e_3^2) = -W. \tag{39}$$

Therefore, similar to the proof of B) of Theorem 1, the results in B) of Theorem 2 can be proved, by applying Barbalat's lemma and the fact that all signals are bounded. □

Remark 2: Results of Theorem 2 indicate that the proposed desired compensation ARC controller (35) has the same performance properties as the previous ARC controller (16). Furthermore, the DCARC law (35) also has the following advantages: (i) Since the regressor φ_{2d} and φ_{3d} depend on the reference trajectory only, it is bounded and can be calculated offline to save on-line computation time if needed. (ii) Gain tuning process becomes simpler since some of the bounds like the bound of the first term inside the bracket of the left hand side of (8) can be estimated offline. (iii) The effect of measurement noise is reduced.

Table 1. Estimated parameters of the verification platform.

Parameters	Value	Parameters	Value
K_F	0.65	B	0.065
J_m	3.2×10^{-4}	N	8.8
J_l	11	K_E	0.56

4. COMPARATIVE EXPERIMENTAL RESULTS

4.1. Experiment setup

To verify above controller design and study fundamental problems in high accuracy tracking control of the TEOGSP system associated with nonlinear friction compensation, a verification platform has been set up. Specifications of initial estimated parameters are listed in Table 1. More details of this platform including system parameters can be found in [17].

4.2. Comparative experimental results

The following five controllers are compared to verify the effectiveness of the proposed control schemes in the next experiments:

1) ARCF: This is the adaptive robust controller (14) with static friction model (3) proposed in this paper. The bounds of parametric ranges are given by $\theta_{\min} = [0, 0, 0, -0.01, 0, 0, 0, 0]^T$, $\theta_{\max} = [5\times10^{-3}, 0.1, 0.4, 0.01, 50, 100, 1, 30]^T$. The initial estimate of θ is chosen as $[1.9\times10^{-3}, 1\times10^{-3}, 0.0353, 0, 0.02, 0, 0, 0]^T$. The bounds of z estimation are chosen as: $z_{\max} = 4\times10^{-3}$, $z_{\min} = -4\times10^{-3}$. Adaptation rates are set at $\Gamma = \mathrm{diag}\{0.1, 0.5, 2, 0.3, 0.5, 600, 0.01, 230\}$. Friction state estimation rates are set at: $\gamma_1 = 1\times10^{-5}$, $\gamma_2 = 1\times10^{-5}$.

2) DCARCF: This is the desired compensation adaptive robust controller (29) proposed in this paper. For fair comparison, all gains used are chosen same as corresponding gains of the ARCF controller.

3) ARC: The ARC controller is implemented same as the proposed ARCF controller but without friction compensation. Other controller parameters of ARC are the same as the corresponding parameters in the ARCF controller.

4) DCARC: The DCARC controller is implemented same as DCARCF but without friction compensation, thus, the gains are chosen same as corresponding gains of the DCARCF controller.

5) PID: The PID controller parameters are $k_P = 12$, $k_I = 6$, $k_D = 0.6$, which represent the P-gain, I-gain and D-gain respectively.

To assess the quality of control algorithms, the maximum tracking error M_e, average tracking error μ,and standard deviation of the tracking error σ defined in [17] are utilized.

To verify the performance of the proposed controllers, the five controllers are tested for a sinusoidal-like motion trajectory $x_{1d} = 10\cos(3.14t)^\circ$. The corresponding tracking errors under five controllers are shown in Fig. 2. The experimental results in terms of performance indexes are given in Table 2. From these results, the proposed DCARCF controller has a better performance than the other four controllers in terms of both transient and final tracking errors; while ARCF also employed the explicit friction model and thus obtained better performance than ARC and PID. Moreover, by using the parameter adaptation as shown in Fig. 3. The final tracking error of DCARCF is reduced almost down to 0.015° while PID has large quadrant glitch (about 0.06°) due to the large friction disturbance during velocity reversal. This illustrates the effectiveness of using the adjustable desired compensation technique which can effectively overcome the friction disturbance in practice.

To further test the performance of the modified LuGre model based friction compensation scheme in the pro-

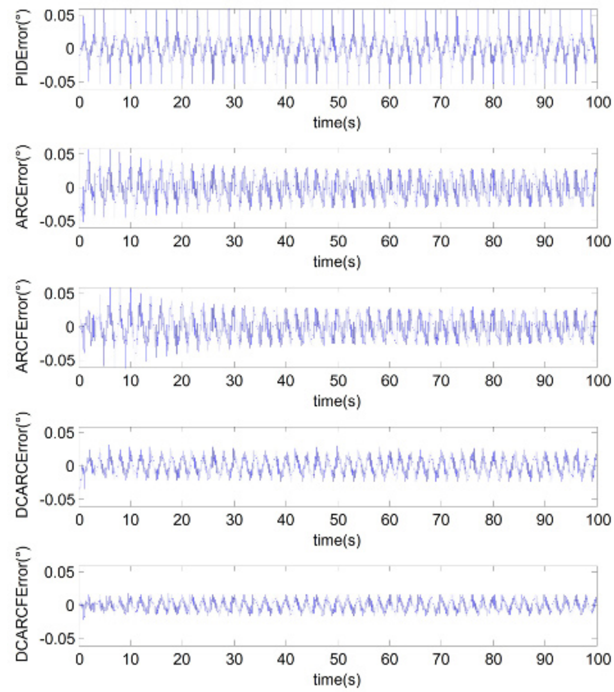

Fig. 2. Tracking errors of five controllers in normal case, PID, ARC, ARCF, DCARC, DCARCF, respectively.

Table 2. Performance indexes during the last two seconds.

Indexes	M_e	μ	σ
PID	0.0581	0.0132	0.0106
ARC	0.0283	0.0125	0.0076
ARCF	0.0260	0.0122	0.0073
DCARC	0.0234	0.0091	0.0046
DCARCF	0.0152	0.0059	0.0034

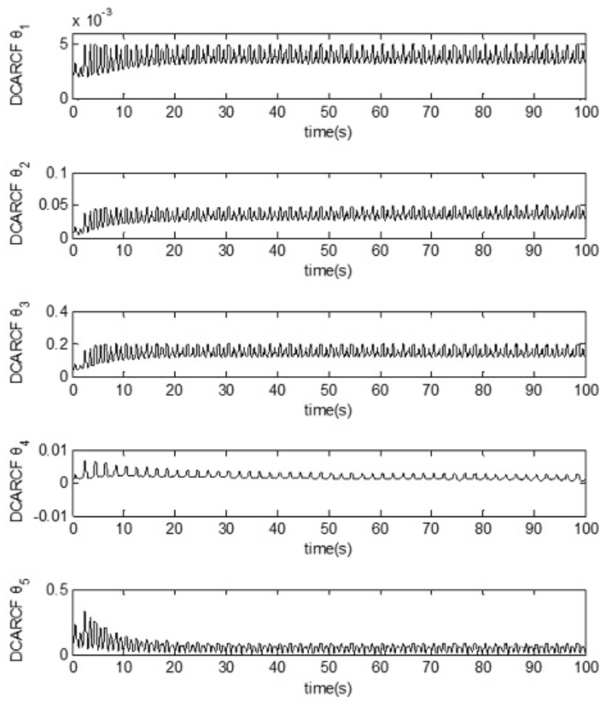

Fig. 3. Parameter estimations of DCARCF controller.

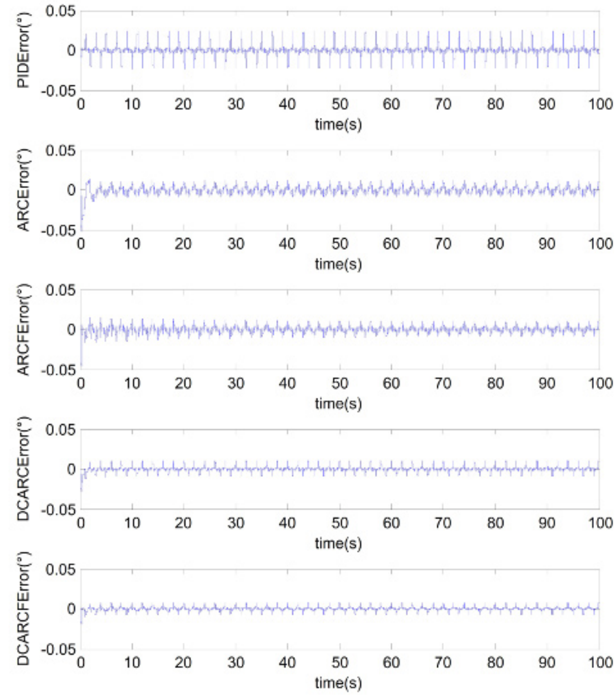

Fig. 4. Tracking errors of five controllers in slow tracking case.

Table 3. Performance indexes during the last two seconds.

Indexes	M_e	μ	σ
PID	0.0247	0.0032	0.0050
ARC	0.0111	0.0032	0.0022
ARCF	0.0099	0.0024	0.0021
DCARC	0.0101	0.0016	0.0023
DCARCF	0.0077	0.0012	0.0014

posed algorithm, in this experimental test, a slow motion trajectory $x_{1d} = \cos(3.14t)]^\circ$ is given. The tracking errors of the five controllers are shown in Fig. 4, respectively. The performance indexes with this case are collected in Table 3. As seen, even for such a slow tracking experiment under strong nonlinear friction, the proposed DCARCF controller is able to compensate the modeled nonlinear friction and attenuate unmodeled effects and an improved performance is achieved in comparison to the other four controllers. The parameter estimation of DCARC are omitted since they are regular and bounded. More clear comparison is presented in Fig. 5. Whereas, parameter adaptation can improve the tracking performance, as done in ARC and DCARC.

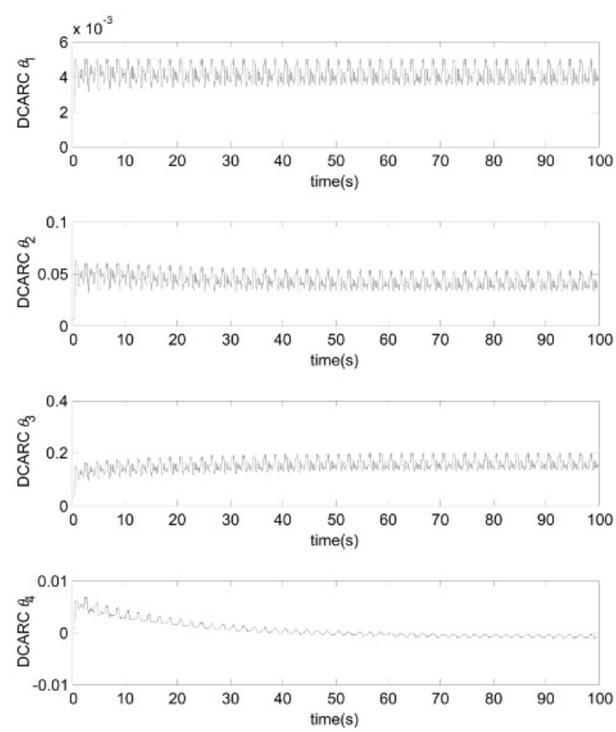

Fig. 5. Parameter estimations of DCARCF controller.

5. CONCLUSION

In this paper, a continuous desired compensation ARC controller have been developed for the TEOGSP systems. By employing the priori bounds on parameter variations and unmeasured friction internal state, discontinuous projection mapping is utilized in learning process to improve the stability of parameter adaptation and friction state estimation. Hence, the synthesized adaptive model compensation and regressor depend on the desired trajectory and parameter estimates only, which facilitates the control gains tuning process and alleviates the effect of measurement noise on the adaptive model compensation. Comparative experimental results will be shown to verify the effectiveness of the DCARC algorithm.

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